International Journal of Theoretical Physics, Vol. 32, No. 11, 1993

Nonlinear Second Harmonic Generation by Light Wave–Plasma Interaction in Oscillating Magnetic Field

Sh. M. Khalil,[1,2] F. Sharaf,[3] and B. F. Mohamed[1,2]

Received September 22, 1992

The nonlinear generation of second harmonic electromagnetic waves in a thin inhomogeneous (dense and rarefied) plasma layer (of length d) by obliquely and normally incident light waves is analyzed. We consider the effect of an external time-dependent magnetic field on the generation and amplification of waves. Two cases are considered, when the magnetic field oscillates at a frequency (i) equal to and (ii) double that of the incident wave. For normal incidence, waves are not radiated in case (i), while in case (ii) the second harmonics are radiated equally from the plasma boundaries at $x = 0$ and $x = d$. For a rarefied plasma, the second harmonics are radiated with equal amplitudes in both cases.

1. INTRODUCTION

Light wave–plasma interaction is of great interest and importance not only from the point of view of reflection or absorption, but also due to other effects, such as breakdown, self-focusing, particle acceleration, trapping, phase modulation of the reflected light, harmonic generation, generation of magnetic fields, and various nonlinear effects and instabilities. Among these, the wave generation due to an incident light wave on an inhomogeneous plasma is one manifestation of the nonlinear properties of the plasma.

The effect of a strong second harmonic signal generation arises as a second-order interaction when the wavelengths in a weakly inhomogeneous plasma are much less than the distance over which the plasma parameters

[1]International Centre for Theoretical Physics, Trieste, Italy.
[2]Permanent address: Plasma Physics Department, Nuclear Research Centre, Atomic Energy Authority, Cairo, Egypt.
[3]Physics Department, Faculty of Science, Suez-Canal University, Ismailia, Egypt.

undergo considerable change, where the dependence of the wave field on spatial coordinates becomes essentially anharmonic. A similar situation also arises in the absence of plasma resonance in the region of a strongly inhomogeneous plasma when the plasma density changes substantially at a distance smaller than or of the order of the wavelengths.

Second harmonics generated by S-polarized (Dolgopolov *et al.*, 1975; Khalil *et al.*, 1984) or P-polarized (El-Siragy *et al.*, 1984) electromagnetic waves incident on a narrow inhomogeneous plasma layer are found to increase sharply in the presence of an external static magnetic field.

In present paper, we consider the generation and radiation of second harmonics due to the nonlinear interaction of an obliquely incident P-polarized light wave ($\mathbf{E} = (E_x, E_y, 0)$, $\mathbf{H} = (0, 0, H_z)$) with an inhomogeneous thin plasma layer of width d under the effect of an external oscillating magnetic field directed in the z-direction, $\mathbf{H}_0 \simeq e_z H_0 e^{-i\omega_m t}$. Two cases will be considered, when (i) $\omega_m = \omega_0$ and (ii) $\omega_m = 2\omega_0$, where ω_0 and k_0 are, respectively, the frequency and wave number of the incident wave.

2. SECOND HARMONIC GENERATION

In the linear approximation and for small perturbation, any value $F(\mathbf{r}, t)$ will be represented by

$$F(\mathbf{r}, t) \simeq f(x) \exp[i(ky - \omega t)]$$

The set of equations used here are the equations of motion, the Maxwell and continuity equations.

Under the conditions of weak nonlinearity, the current density components of generated waves with frequency $2\omega_0$ are

$$\mathsf{J}^G_{(x,y)} = i\frac{\omega_{pe}^2}{8\pi\omega_0}\mathsf{E}^G_{(x,y)} + i\mathsf{J}'_{(x,y)} + i\alpha\frac{\omega_c}{2\omega_0}\mathsf{J}_{(x,y)} \tag{1}$$

where ω_c is the electron cyclotron frequency. Relation (1) is obtained for $\omega_m = \omega_0$, and a similar relation with $\omega_c = 0$ can be obtained for $\omega_m = 2\omega_0$. Here ω_{pe} is the Langmuir frequency. $\mathsf{J}'_{(x,y)}$ represent the fundamental electric current density as

$$\mathsf{J}'_{(x,y)} = \frac{\alpha e}{2m\omega_0 C}\mathsf{J}_{(y,x)}H_z + \frac{1}{e\omega_0}\left[\left(\frac{\mathsf{J}\cdot\nabla}{2} + \nabla\cdot\mathsf{J}\right)\frac{\mathsf{J}}{n_0}\right]_{(x,y)}$$

where $\alpha = -1, 1$ for the x and y components, respectively. The current given by relation (1) can generate only waves which are P-polarized. The generated electric field components due to these currents are derived as follows (at $\omega_m = \omega_0$):

$$\mathsf{E}_x^G = -\frac{N}{\epsilon}\,\mathsf{H}_z^G + \frac{2\pi}{\omega_0\epsilon}\,\mathsf{J}_x' - \frac{\pi}{\omega_0\epsilon}\left(\frac{\omega_c}{\omega_0}\right)\mathsf{J}_y \tag{2}$$

$$\mathsf{E}_y^G = -i\frac{c}{2\omega_0\epsilon}\frac{\partial}{\partial x}\,\mathsf{H}_z^G + \frac{2\pi}{\omega_0\epsilon}\,\mathsf{J}_y' + \frac{\pi}{\omega_0\epsilon}\left(\frac{\omega_c}{\omega_0}\right)\mathsf{J}_x \tag{3}$$

where $\epsilon = 1-(\omega_{pe}/2\omega_0)^2$. When $\omega_m = 2\omega_0$ the generated electric field components are given also by (2)–(3) with $\omega_c = 0$.

The generated magnetic field component of the P-polarized wave at double the frequency of the incident wave is governed by the following inhomogeneous differential equation:

$$\epsilon\frac{\partial}{\partial x}\left(\frac{1}{\epsilon}\frac{\partial}{\partial x}\,\mathsf{H}_z^G\right) + X^2\mathsf{H}_z^G = \mathscr{R}_0(x) + \mathscr{R}(x) \tag{4}$$

where $X^2 = k_0^2(\epsilon/N^2 - 1)$, $N = k_0C/\omega_0$, $\mathscr{R}_0(x)$ is the nonlinear source term, while $\mathscr{R}(x)$ is due to the effect of the external magnetic field. These functions are given by

$$\mathscr{R} = (\omega_c/\omega_0)\mathscr{R}_h(x) \qquad \text{at} \quad \omega_m = \omega_0$$

$$\mathscr{R} = -\epsilon(2\omega_0/C)^2 H_0 \qquad \text{at} \quad \omega_m = 2\omega_0$$

$$\mathscr{R}_0 = -\frac{4\pi}{C}\left(2k_0\mathsf{J}_x' + i\epsilon\frac{\partial}{\partial x}\frac{1}{\epsilon}\,\mathsf{J}_y'\right)$$

$$\mathscr{R}_h = \frac{2\pi}{C}\left(2k_0\mathsf{J}_y + i\epsilon\frac{\partial}{\partial x}\frac{1}{\epsilon}\,\mathsf{J}_x\right)$$

Solutions of (4) in vacuum ($x \ll 0, x \gg d$) and in the plasma inhomogeneous layer can be obtained by using the methods of successive approximation taking into account that in the layer region ($0 \le x \le d$), the plasma density and consequently its dielectric permeability ϵ are sharply changed with the variation of x, i.e., $|\partial\epsilon/\partial x| \cong \epsilon/d \gg k_0|\epsilon|$, which allows us to use the condition $|Xd| \ll 1$. Accordingly, we obtain

$$\mathsf{H}_z^G = \mathsf{A}_0 e^{iX_0x} + \mathscr{H} \qquad \text{at} \quad x \le 0 \tag{5}$$

$$\mathsf{H}_z^G = \mathsf{A}_d e^{iX_0(x-d)} + \mathscr{H} \qquad \text{at} \quad x \ge d \tag{6}$$

$$\mathsf{H}_z^G = \mathsf{A}_0\left[1 - \int_0^x \epsilon\,dx' \int_0^{x'} (X^2/\epsilon)\,dx'' + iX_0\int_0^x \epsilon\,dx'\right] + \int_0^x \epsilon\,dx' \int_0^x (\mathscr{R}_0/\epsilon)\,dx'' + \mathsf{M}(x) \qquad \text{at} \quad 0 \le x \le d \tag{7}$$

A_0 and A_d are the amplitudes of the generated magnetic field at $x = 0$ and d, respectively, and:

At $\omega_m = \omega_0$:

$$\mathscr{H} = 0, \qquad \mathsf{M}(x) = (\omega_c/\omega_0) \int_0^x \epsilon \, dx' \int_0^{x'} (\mathscr{R}_h/\epsilon) \, dx''$$

At $\omega_m = 2\omega_0$:

$$\mathscr{H} = -(2\omega_0/X_0\epsilon)^2 H_0$$

$$\mathsf{M}(x) = -(2H_0/N_0)\left[1 - \int_0^x \epsilon \, dx' \int_0^{x'} (X^2/\epsilon) \, dx'' + X_0^2 \int_0^x x'\epsilon \, dx'\right]$$

From (5)–(7) and making use of the continuity of the functions $\mathsf{H}_2^{\mathcal{G}}$ and $(\partial/\partial x)\mathsf{H}_2^{\mathcal{G}}$ at $x = 0$, we can obtain the amplitudes of the generated second harmonics as

$$\mathsf{A}_0 = (1/S)\left[\int_0^d (\mathscr{R}_0/\epsilon) \, dx + iX_0 \int_0^d \epsilon \, dx \int_0^x (\mathscr{R}_0/\epsilon) \, dx'\right] + B_1 \tag{8}$$

$$\mathsf{A}_d = A_0\left(1 + iX_0 \int_0^d \epsilon \, dx\right) + \int_0^d \epsilon \, dx \int_0^x (\mathscr{R}_0/\epsilon) \, dx' + \mathsf{B}_2 \tag{9}$$

where

$$S = -i \cdot 2X_0 + X_0^2 \int_0^d \epsilon \, dx + \int_0^d (X^2/\epsilon) \, dx$$

and

$$\mathsf{B}_1 = (1/S)(\omega_c/\omega_0)\left[iX_0(\omega_0/\omega_c)\mathsf{B}_2 + \int_0^d (\mathscr{R}_h/\epsilon) \, dx\right] \quad \text{at} \quad \omega_m = \omega_0 \tag{10}$$

$$= -(H_0/S)(2/N)^2\left[iX_0^3 \int_0^d x\epsilon \, dx + X_0^2 d - \int_0^d \epsilon \, dx \int_0^x (X^2/\epsilon) \, dx' - \int_0^d (X^2/\epsilon) \, dx\right] \quad \text{at} \quad \omega_m = 2\omega_0 \tag{11}$$

$$\mathsf{B}_2 = (\omega_c/\omega_0) \int_0^d \epsilon \, dx \int_0^x (\mathscr{R}_h/\epsilon) \, dx' \quad \text{at} \quad \omega_m = \omega_0 \tag{12}$$

$$= -H_0\left[X_0^2 \int_0^d x\epsilon \, dx - \int_0^d \epsilon \, dx \int_0^x (X^2/\epsilon) \, dx'\right] \quad \text{at} \quad \omega_m = 2\omega_0 \tag{13}$$

It is clear from (8)–(13) that waves are generated at second harmonics from the inhomogeneous plasma layer, with different amplitudes. It is also shown that an external magnetic field has two effects on the wave generation, (1) amplification of the generated second harmonic when it oscillates at a frequency equal to that of the incident light wave ($\omega_m = \omega_0$), and (2) a reduction (damping) of the generated wave when it oscillates at frequency $\omega_m = 2\omega_0$, provided that $\mathsf{B}_1, \mathsf{B}_2 > 0$.

3. EFFECT OF LOW-DENSITY PLASMA AND NORMAL INCIDENCE

In the case of rarefied plasmas, $\omega_{pe} \ll \omega_0$, we can obtain the following expression for the generated wave amplitudes:

$$\mathsf{A}_0 = \mathsf{A}_d = (i/2X_0)\left[\int_0^d \mathcal{R}_0|_{\epsilon=1}\, dx + (\omega_c/\omega_0)\int_0^d \mathcal{R}_h|_{\epsilon=1}\, dx\right] \qquad (14)$$

Relation (14) is valid for the case of $\omega_m = \omega_0$, while for the case $\omega_m = 2\omega_0$, a similar relation can be obtained with $\omega_c = 0$, i.e., the magnetic field has no effect in this case. (14) shows that the generated second harmonics are radiated equally from the plasma boundaries at $x = 0, d$. For static external magnetic field (El-Siragy *et al.*, 1984), waves are radiated from a low-density plasma with different amplitudes.

Let us consider also the case of normal incidence of light waves, i.e., when $k_0 = 0$, we get

$$\mathsf{A}_0 = \mathsf{A}_d = 0 \qquad \text{at} \quad \omega_m = \omega_0 \qquad (15)$$

$$\mathsf{A}_0 = \mathsf{A}_d = 2(\omega_0/c)\left[\int_0^d x\epsilon\, dx - (1/1 - N^2)\int_0^d \epsilon\, dx \int_0^x (1 - N^2/\epsilon)\, dx'\right]H_0 \qquad \text{at} \quad \omega_m = 2\omega_0 \qquad (16)$$

From (15) and (16) we see that, at $\omega_m = \omega_0$, second harmonics are not emitted by P-polarized incident light, while at $\omega_m = 2\omega_0$ the second harmonics are again emitted with large amplitude proportional to the strength of the applied magnetic field. In general, in the presence of an oscillating magnetic field, second harmonics are not radiated from the plasma layer when we consider both normal incidence and a rarefied plasma.

Since we considered here high-frequency oscillations ($\omega_m = \omega_0, 2\omega_0$), the spatial inhomogeneity of the external magnetic field may be also taken into account. This effect will cause more amplification (at $\omega_m = \omega_0$) or more damping (at $\omega_m = 2\omega_0$) of the generated waves. This case will be investigated in more detail in due course.

ACKNOWLEDGMENTS

Two of the authors (S.M.K. and B.F.M.) would like to thank Prof. Abdus Salam, the International Atomic Energy Agency, and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste. Thanks are also due to Prof. M. Hassan, TWAS, Trieste, for support and advice.

REFERENCES

Dolgopolov, V. V., El-Siragy, N. M., and Sayed, Y. A. (1975). In *7th European Conference on Controlled Fusion and Plasma Physics*, Lausanne.

El-Siragy, N. M., Khalil, Sh. M., Amein, W. H., and Nagy, O. Z. (1984). *Egyptian Journal of Physics*, **15**, 213.

Khalil, Sh. M., El-Siragy, N. M., Sayed, Y. A., and El-Sherif, R. N. (1984). *Physica Scripta*, **30**, 357.